

## Scale independence of rock physics transforms

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### Summary

Rock properties, such as porosity, permeability, and elastic moduli, depend on the scale (volume) of measurement. They may spatially vary within millimeters and, certainly, within inches and feet. This natural variability renders meaningless the concept of a data point, simply because the datum will be different just a few inches away, or if averaged in a larger volume. However, some transforms from one property to another appear scale independent. Obtained at a microscale, they can be potentially applied at a larger scale in the field. This makes us believe that such transforms, stationary with respect to position and scale, should replace single-category properties (e.g., porosity) as the primary attributes of rock.

### Introduction

Consider the Finney (1970) pack, a dense random pack of identical spheres, arguably one of the most homogeneous granular objects (Figure 1). This pack is comprised of about 4000 grains. Its porosity is about 0.36.

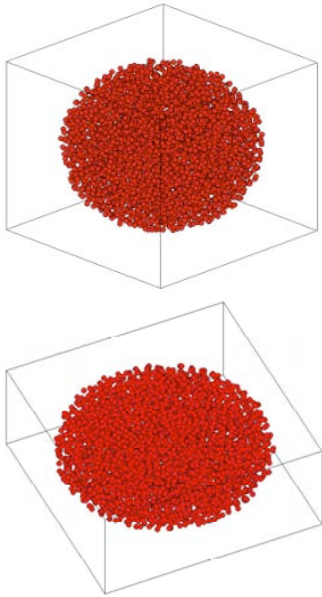


Figure 1: The Finney pack, a dense random pack of identical steel spheres physically assembled with the coordinates of each sphere center consequently measured. This pack is shown here in two different perspectives. The grains are red.

Let us select a cube sub-volume at the center of this pack and further (evenly) divide this cube into eight sub-cubes

(Figure 2). The porosity of these sub-volumes plotted in the same figure is not stationary in space: depending on the subsample, it can be as small as 0.32 or as large as 0.41, whereas the porosity of the larger volume is 0.36. We can reduce the porosity of the original pack by digitally expanding the radius of each sphere to 1.05 and 1.10 of its original size while keeping their centers fixed in space. Spatial non-stationarity of porosity persists in these altered samples (Figure 3).

A question then is how the porosity (or any other rock property) measured at a given scale, be it in the lab or in the well, is relevant to forecasting and understanding processes at another scale (e.g., during reservoir simulation).

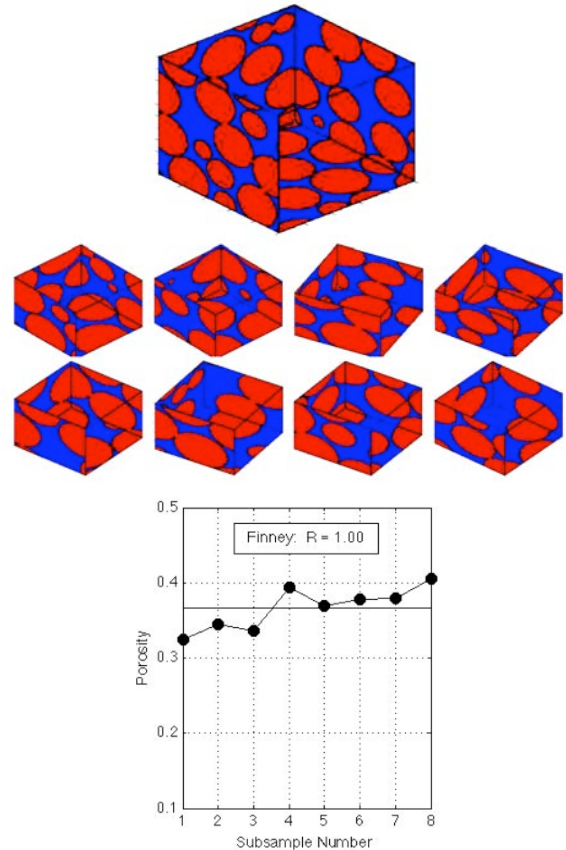


Figure 2: Cube volume of the Finney pack (top) and its eight subsamples (bottom). The grains are red while the pores are blue. The bottom frame shows the porosity of the eight subsamples (connected symbols). The horizontal line is the porosity of the larger cube.

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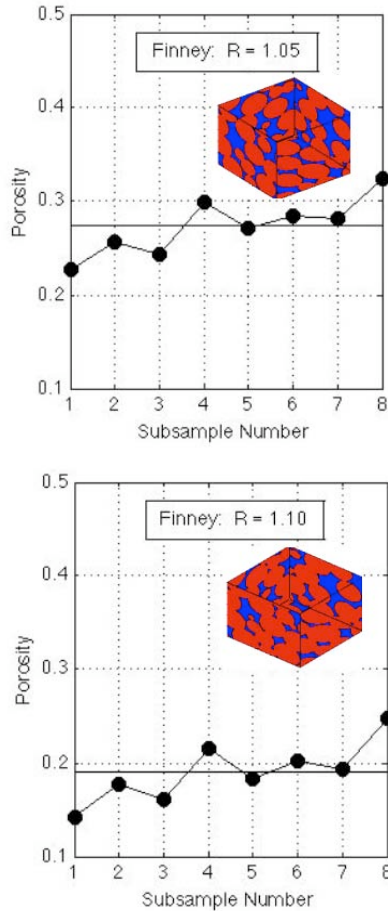


Figure 3: Porosity of eight subsamples of a larger cube obtained from the Finney pack by expanding each sphere. The radius of each sphere is 1.05 (top) and 1.10 (bottom) of the original radius. The horizontal lines are for the porosities of the larger cubes.

One may argue of course, that the spatial non-stationarity can be alleviated by simply selecting a volume large enough, the so-called elementary representative volume (REV). This will certainly be the case in the Finney pack.

We dispute this point by speculating that there is no REV to be found in natural rock (Figure 4 and 5) which is heterogeneous at all scales. This is demonstrated by many well data where rock properties can vary appreciably between two points just inches apart.

Therefore, it may not be valid to assign a single porosity, permeability, resistivity, or velocity to a volume in the subsurface. What then should replace these traditional rock physics attributes?

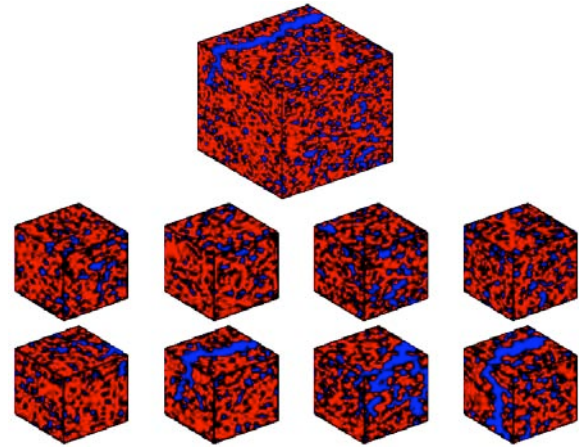


Figure 4: Subsampling of a natural oil sand. The larger image (top) is a high-resolution 3D CT-scan subsequently segmented to delineate pores (blue) and mineral matrix (red). The side of this larger cube is about 3 mm. The eight subsamples evenly cut from this cube are shown in the bottom.

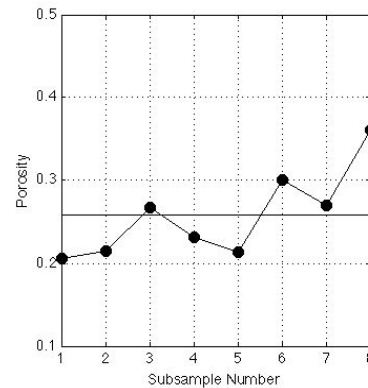


Figure 5. The porosity of the eight subsamples of the oil sand from Figure 4 (connected symbols). The porosity of the larger cube is the horizontal line.

### Using Trends instead of Data Points: Permeability

It is reasonable to assume that if porosity is not stationary in space, neither will be the corresponding permeability. To measure permeability on samples and subsamples shown in Figure 2 and 4 is arguably impossible in the physical laboratory.

However, it is definitely possible in the digital laboratory by simulating the Navier-Stokes flow through the digital pore space either constructed on the computer, as in the Finney pack example, or imaged, as in the oil sand in Figure 4. Such computer experiments have been robust and

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repeatable (Dvorkin et al., 2008; and Dvorkin and Nur, 2009).

Let us first use a much simpler method for permeability estimate by directly, voxel-by-voxel, calculating the porosity ( $\phi$ ) and specific surface area ( $s$ ) of the original and altered Finney pack samples and their subsamples and then relating these two quantities to the absolute permeability ( $k$ ) through the Kozeny-Carman equation

$$k \sim (\phi - \phi_p)^3 / s^2,$$

where  $\phi_p$  is the percolation porosity (Mavko and Nur, 1997) at which the pore space becomes disconnected and rock impermeable.

The absolute permeability thus obtained for the three larger cubes from the Finney pack (for the original grain radius and the grain radii 1.05 and 1.10 of the original) is plotted versus the porosity of these samples in Figure 6a. Next, we add to this plot the permeability versus porosity for the eight subsamples of each of these three samples (Figure 6b, 24 additional data points).

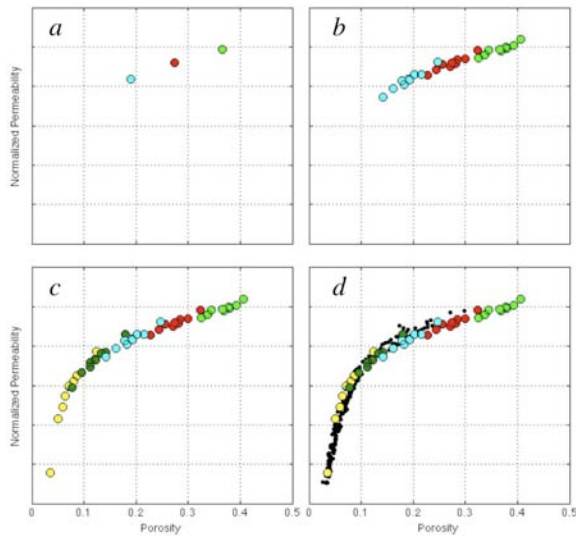


Figure 6: Normalized permeability versus porosity in digital and physical samples. Color symbols are for digital samples obtained by altering and subsampling the Finney pack as described in the text. Smaller black symbols are the Fontainebleau data. A. Three Finney pack cubes, the original one and two altered to achieve smaller porosity. B. 24 subsamples of these three samples added. C. Two more altered Finney pack samples and their subsamples. D. Same with Fontainebleau data on top. The permeability of the synthetic samples is normalized to approximately match that of Fontainebleau sandstone at high porosity.

Notice that these subsample permeability-porosity data pairs fall on the trend formed by the first three data pairs

obtained for the larger samples. A permeability-porosity trend based on subsampling mimics that based on alteration of a sample.

We facilitate this point by adding to the plot our results for two more larger cubes of the same portion of the Finney pack, with the grain radii 1.15 and 1.20, respectively, of the original radius. The permeability-porosity pairs for these two samples and their sixteen subsamples displayed in Figure 6c conform with the trend formed by the first three samples and their 24 subsamples.

In Figure 6d, the experimental Fontainebleau sandstone trend (Bourbie and Zinsner, 1985) is plotted on top of these synthetic data. The apparent match between the physical and synthetic trends makes us conclude that a permeability-porosity transform obtained at a millimeter scale on digital samples is valid at a core-plug scale and in physical samples.

Figure 7 presents digital permeability-porosity data for millimeter-scale Berea sandstone and oil sand samples (the latter shown in Figure 4). These samples were CT-scanned at high resolution and then segmented to separate the pores from the mineral. The porosity was obtained by counting the voxels inside the pore space. The absolute permeability was computed by simulating slow viscous flow in the pore space (in all its complexity and without any idealization of the geometry) using the Lattice-Boltzmann digital method.

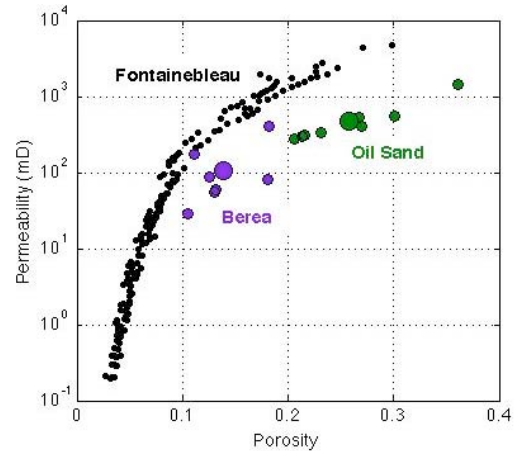


Figure 7: Permeability versus porosity computed for oil sand (green) and Berea sandstone (magenta). Large color symbols are for the actual digital samples while smaller color symbols are for their subsamples (eight for each sample). Black symbols are the Fontainebleau data of Bourbie and Zinsner (1985).

Physical laboratory datum was not available for the oil sand. In Berea, the digital porosity and permeability closely matched the physical measurements.

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The most remarkable feature of the cross-plot in Figure 7 is not the realistic values of the computed permeability but the permeability-porosity trend formed by the data pairs computed on the eight subsamples of a larger sample. This trend appears especially tight for the oil sand.

Once again we observe that although porosity and permeability are not stationary in space, a sustained trend between these two properties exists. Moreover, this trend can be obtained just from a single and very small physical sample by imaging and computing.

Because the cases under examination imply spatial and cross-scale stationarity of a trend between two rock properties, we propose that such trends be used instead of data points. Such trends should replace (or at least complement) traditionally used data points not only because of their persistence but also because by using them we can address an as yet unresolved problem: how to utilize disparate measurements obtained at different scales and by different techniques to predict rock properties from remote sensing in the field.

### Elastic Properties

Using trends in remote sensing cannot be complete without examining the elastic properties of rock and their relation to the bulk and transport properties. The question posed in relation to the elastic properties is the same as addressed earlier in this paper in relation to the transport properties: will trends obtained at a microscale be spatially stationary and hold at a core-plug scale?

Results displayed in Figure 8 indicate that the answer to this question is positive, at least for some clean (clay-free) sandstone samples. The light-blue triangles in this plot are from laboratory measurements on room-dry samples at high confining pressure. The red symbols represent the elastic moduli computed on CT-scanned microsamples of similar rock formation. Each microsample was divided into eight subsamples and the elastic moduli were computed for each of these subsamples as well.

The digital elastic moduli computed just a few mm apart differ from each other. However, if plotted versus porosity, they form a trend that is spatially stationary and also matches the laboratory trend, which means that it holds at different scales. Similar results, but for carbonates, are shown in Figure 9.

### Conclusion

Heterogeneity of natural rock persists at many scales. Then how can we use data obtained on a set of samples in the context of remote sensing, which probes different and

differently-sized volumes within a formation? One answer is that pairs of datapoints obtained on an internally heterogeneous dataset form a trend that is valid over a range of scales and can be possibly applied to a remotely-sensed quantity to infer another desired property at the scale of practical measurement.

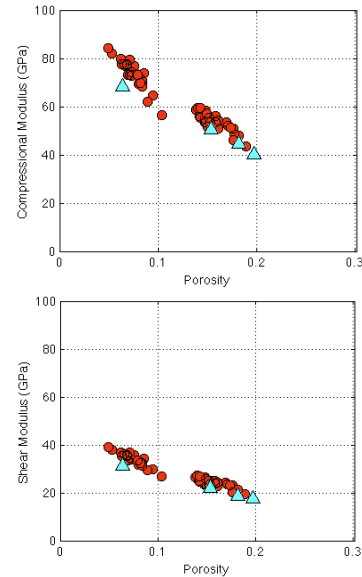


Figure 8: Elastic moduli versus porosity for clean sandstone. Triangles are laboratory data. Red circles are mm-scale digital calculations on a few CT-scanned samples of similar rock and their subsamples.

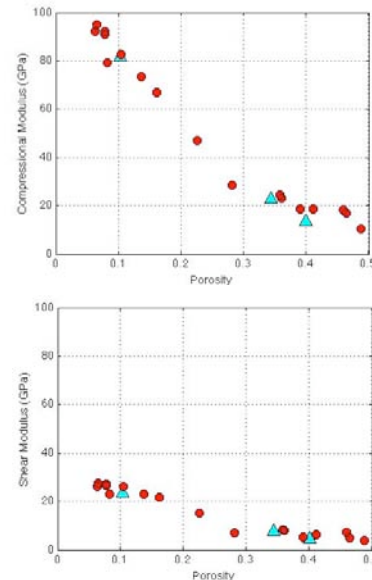


Figure 9: Same as Figure 8 but for carbonate samples.